Understanding Guitar Intonation

The tones on a guitar, being a fretted string instrument, are pitched by shortening individual strings against the frets. As the strings are pressed against the frets they are essentially deformed and the tension in them increases, causing certain intonation issues. This document will help you understand the best way of tuning your guitar.

Scale Length Calculation

The guitar scale length, or open string length, is designed in proportion to the length of the guitar player's fingers. The standard scale of the classical guitar is 650 mm. However, if the span between the thumb and little finger is equal to or less than 22.5 cm, a shorter scale of, let's say, 640 mm is more appropriate regarding playing technique and sound formation.

Once the scale is set, we can proceed to calculate the spacing between frets. The basic formula for fret spacing calculation is as follows: $\frac{S}{c} = f_1$

Where: S = Scale length

c = constant $\sqrt[12]{2} = 1{,}059463094$

f_1 *= distance from the 1st fret to the bridge saddle*

By substituting the calculated value in the basic formula: $\frac{f_1}{c} = f_2$*, we obtain the distance of the second fret from the bridge. The same method is applied to calculate the remaining fret positions.*

The nut to fret distance can be calculated as follows: S - $f_n = x_n$

Where: S = Scale length

f_n *= fret no.*

x_n *= fret distance from the nut*

The values calculated in this manner are, however, only theoretical. If the guitar were to function, for example, as a monochord, in which case the string is shortened by shifting the zero fret, these values could be utilized. This is because by shifting the zero fret the tone pitch is changed without string deformation. It's as if you were constantly playing an open string

with only the tone pitch changing. It's somewhat different when playing the guitar, because the string is being shortened by placing a finger on the fingerboard just behind the fret (excluding open strings). This creates a different tension than if you were to play it like an open string. The tension will naturally vary in different positions. For all guitar strings to be truly in tune, the guitar must be compensated.

How to Correctly Compensate the Guitar

There is a number of known methods of dealing with guitar intonation issues. However, it is important to consider whether a given method actually does resolve the intonation issues and, if so, to what extent. Let us now examine several ways in which a guitar can be compensated. A word should first be said about what happens if the guitar is not compensated at all. The guitar is constructed based on a mathematical model where the 12th position is exactly half of the scale. All of the tones played on the fingerboard will be higher and the deviation will deepen towards the bridge, for the string height above the fingerboard increases just as the string tension is increased. This why every guitar needs to be compensated. To facilitate better understanding I'll endeavor to articulate the issue via the use of charts in which several known compensation methods will be addressed. However, they should only be used as reference charts because dimensions may vary from instrument to instrument. Configurations, such as string height above the fingerboard (the action), fret height or tension and string type, may be specific for each instrument.

Chart 1 *shows a traditional, most commonly used method. The open string is tuned to its octave (12th position) by shifting the bridge saddle separately for each string.*

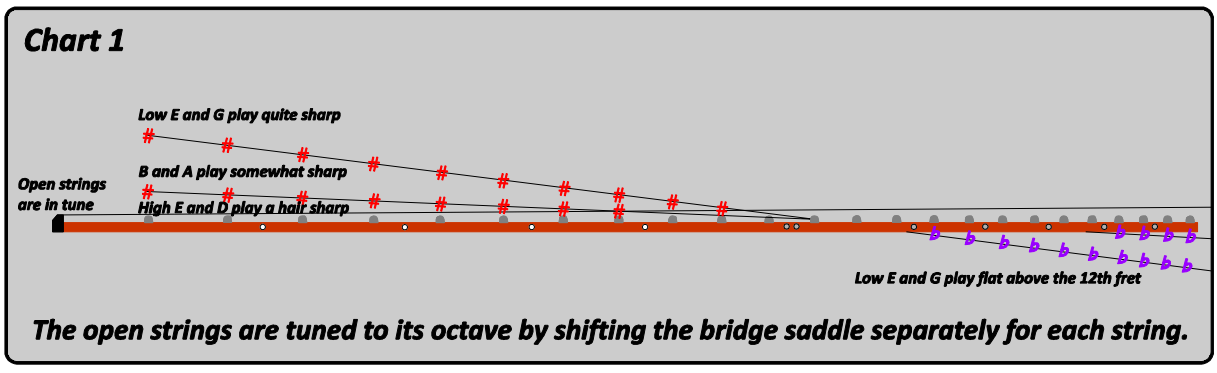


Chart 1 demonstrates that if all open strings are tuned to the octave by shifting the bridge saddle, the fingerboard positions are all out of tune.

On the other hand, ***Chart 1a*** *shows a compensation of a saddle which has not been tuned for each string separately, but a compromise made between all strings has been used instead.*

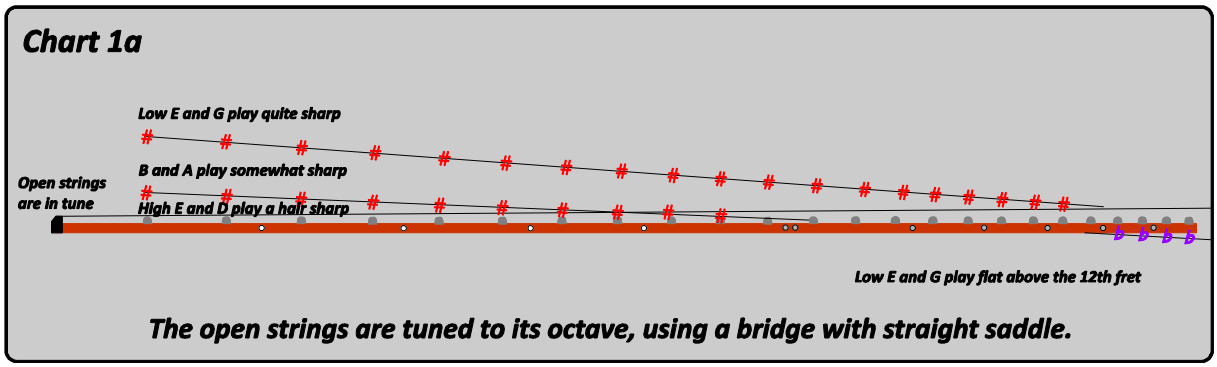
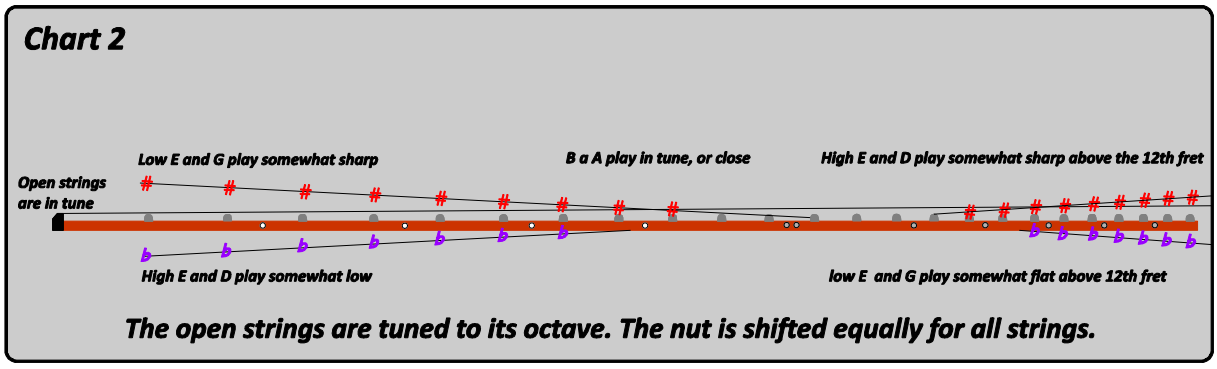
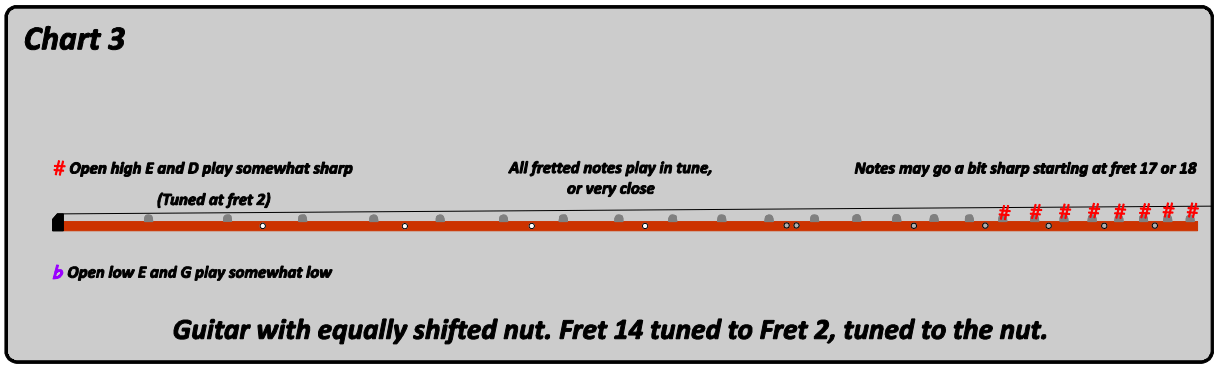


Chart 2 *essentially replicates the variation presented in Chart 1 with the addition of nut compensation towards the bridge. As a compromise, the nut is shifted equally for all strings.*

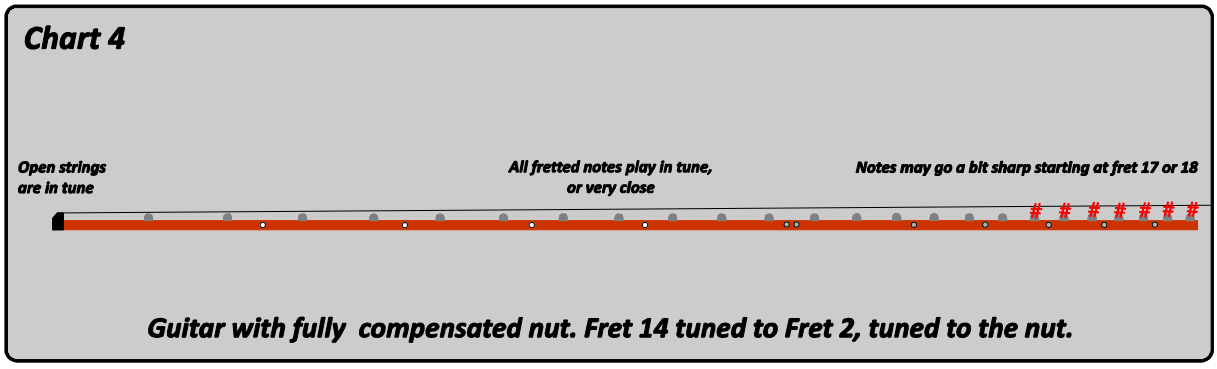


The chart above shows improvement in intonation but points out issues that remain unresolved after the nut is moved forward, as the graph illustrates. Even though this intonation method reduces the maximum disharmony between open strings and notes played on the fingerboard, the intonation error between notes played on the fingerboard remains.

Chart 3 *presents a method whereby we do not tune open strings to the octave, but rather tune the 2nd position to the* 14^{th} *position. What I mean by this is that, for example, F sharp is tuned on the first string (on the fingerboard) and the bridge saddle is compensated by adjusting it to the octave, i.e. the* 14^{th} *position (F sharp). The trick lies in not fine-tuning* an open string to the 12th position on the fingerboard *but, instead, compensating fingerboard tones with fingerboard tones, while the nut is shifted forward. Those who rarely play with open strings will find this setting perfect, the only downside being that the guitar must be tuned on the fingerboard (instead of open strings).*



And finally, ***Chart 4*** *presents a method involving a compensated bridge saddle with the fingerboard and a fully compensated nut.*



As Chart 4 shows, the tones on the finger board are perfectly in tune even when open strings are tuned. Only the tones played behind the 17th fret may sound slightly higher. But let's call a spade a spade, when juxtaposed with the standard method, as presented in Chart 1, this result is beyond compare. Here is a brief explanation of approaching the method of a fully compensated nut:

Before anything else, the guitar is tuned in the 2nd position and the bridge saddle compensated so that the 2nd position is in perfect tune with the 14th position. The nut can now be compensated to the 2nd position. That is to say, the 2nd position is tuned once again and the nut compensated until the 2nd position is in tune with the open string.

I hope I've been able to shed at least some light on the issues involved in the tuning of fretted string instruments. It's a pity that nowadays a fully compensated nut cannot be taken for granted in the majority of factory-made instruments and, unfortunately, sometimes even in hand-crafted ones.

Petr Čehovský, 2015